The role of g-wave pairing and Josephson tunneling in high-T_c cuprate superconductors.

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Abstract

The implications of the two-pocket Fermi surface for macroscopic quantum phenomena are considered. We demonstrate that in the case of the two-pocket Fermi surface the g-wave pairing is closely related to the d-wave one. As a result two macroscopic condensates arise. The Josephson tunneling for such two-component system has very special properties. We prove that the presence of the g-wave does not contradict the existing experimental data on tunneling. We also discuss the possible ways to experimentally reveal the g-wave component.

1 Introduction

There is a lot of controversy about the shape of the Fermi surface in cuprate superconductors. In the early days it was believed that it is a small Fermi surface of the doped Mott insulator [1]. Later many of the results from photoelectron spectroscopy (PES) have been interpreted as favoring a large Fermi surface in agreement with Lattinger's theorem [2]. On the other hand most recent PES data[3, 4, 5, 6] once more give indications of a small Fermi surface for underdoped samples. In the present paper we consider the scenario with a small Fermi surface consisting of hole pockets around $(\pm\pi/2, \pm\pi/2)$, see Fig.1a. It is widely believed that the $t - J$ model describes the main details of the doped Mott insulator. To fit the experimental hole dispersion one needs to extend the model introducing additional hopping matrix elements t', t'' (see. e.g. Refs.[7, 8, 9]), but basically it is the $t - J$ model. Superconducting pairing in the $t - J$ model induced by spin-wave exchange has been considered in the papers [10, 11]. It was demonstrated [10] that there is an infinite set of solutions for the superconducting gap. All the solutions have nodes along the lines $(1, \pm 1)$, see Fig.1a. (It is very convenient to use the magnetic Brillouin zone, but it can certainly be mapped to the full zone.) Using translation by the vector of inverse magnetic lattice the picture can be reduced to two hole pockets centered around the points $(\pm\pi/2, \pi/2)$, see Fig.1a. The superconducting pairing is the strongest between particles from the same pocket, and the lowest energy solution for the superconducting gap has only one node line in each pocket. Having this solution in a single pocket one can generate two solutions in the whole Brillouin zone taking symmetric or antisymmetric combinations between pockets. The symmetric combination corresponds to the d-wave (Fig.1b), and the antisymmetric combination corresponds to the g-wave (Fig.1c) pairing. We would like to note that the possibility to generate new solutions taking different combinations between pockets was first demonstrated by Scalettar, Singh, and Zhang in the paper [12]. The energy splitting between the d- and g-wave solutions has been investigated numerically in the Ref.[11]. The g-wave solution disappears and only d-wave one survives as soon as the hole dispersion is degenerate along the face of the magnetic Brillouin zone. Actually in this situation there are no pockets and one has a large open Fermi surface at an arbitrary small hole concentration. However for small well separated pockets the d- and g-wave solutions are almost degenerate. In pure $t - J$ or $t - t' - t'' - J$ model the d-wave solution

always has the lower energy. However if one extends the model including nearest sites hole-hole Coulomb repulsion the situation can be inverted. The nearest sites repulsion does not influence the g-wave pairing and substantially suppresses the d-wave pairing. So it is quite possible that the real ground state has the g-wave superconducting gap.

2 Ginzburg-Landau free energy

In the present paper we consider the scenario with hole pockets well separated in k-space. According to the described above microscopic picture we should consider simultaneously the d- and the g-wave pairing. In general terms such situation was considered a long time ago for conventional superconductors by Leggett [13]. Let us formulate an effective Ginzburg-Landau (GL) theory. In the first approximation we can neglect the interaction between pockets and introduce two macroscopic condensates corresponding to the pockets, $\Psi_1 = |\Psi_1| e^{i\phi_1}$, $\Psi_2 = |\Psi_2| e^{i\phi_2}$, where $|\Psi_1| = |\Psi_2| = |\Psi| = \sqrt{N_h}/2$, N_h is the number density of the condensate holes. The Ginzburg-Landau free energy of a such system in an external magnetic field can be written as

$$
F_0 = \int \left(\frac{1}{2m^\star} |(\nabla - \frac{2ie}{\hbar c}\mathbf{A})\Psi_1|^2 + \frac{1}{2m^\star} |(\nabla - \frac{2ie}{\hbar c}\mathbf{A})\Psi_2|^2 - \right. \tag{1}
$$
$$
\left. a|\Psi_1|^2 + b|\Psi_1|^4 - a|\Psi_2|^2 + b|\Psi_2|^4 + \frac{B^2}{8\pi} \right) dV.
$$

A small interaction between pockets can be described by adding to the free energy (1) the term

$$
F_{\text{int}} = \gamma \int \left(\Psi_1^* \Psi_2 + \Psi_1 \Psi_2^* \right) dV \to 2\gamma V |\Psi_1| |\Psi_2| \cos(\phi_1 - \phi_2), \tag{2}
$$

where γ is a small parameter of the interaction, $\gamma \ll a$. The total bulk energy of the cuprate superconductor equals $F_V^{dg} = F_0 + F_{\text{int}}$, and the equilibrium values of the order parameters are

$$
|\Psi_1|^2 = |\Psi_2|^2 = |\Psi|^2 = (a + |\gamma|)/2b \approx a/2b. \tag{3}
$$

The ground state phase difference $\Delta\phi = \phi_2 - \phi_1$ is determined by the sign of γ: if $\gamma > 0$, then $\Delta\phi = \pi$ (g-wave); if $\gamma < 0$, then $\Delta\phi = 0$ (d-wave). It is also convenient to introduce the d- and g-wave condensates $\Psi_d = \Psi_1 + \Psi_2 = 2|\Psi|\cos(\Delta\phi/2)e^{i\phi}$ and $\Psi_g = \Psi_1 - \Psi_2 = 2|\Psi|\sin(\Delta\phi/2)e^{i\phi-i\pi/2}$, where $2\phi = \phi_1 + \phi_2$. So the ground state has either d- or g-wave symmetry: if $\gamma > 0$ then $\Psi_d = 0, \Psi_g \neq 0$, if $\gamma < 0$ then $\Psi_g = 0, \Psi_d \neq 0$.

3 Tunnel junction

Let us consider a Josephson tunnel contact of a conventional s-wave superconductor (the order parameter $\Psi_s = |\Psi_s| \exp(i\phi_s)$) and a high-$T_c$ cuprate superconductor. Constant supercurrent $\mathbf{j}$ is maintained through the contact. The bulk free energy of the conventional superconductor equals

$$
F_V^s = \int \left(\frac{\hbar^2}{2m_s^\star} \left| (\nabla - \frac{2ie}{\hbar c}\mathbf{A})\Psi_s \right|^2 - a_s |\Psi_s|^2 + b_s |\Psi_s|^4 + \frac{B^2}{8\pi} \right) dV, \tag{4}
$$

and the surface free energy related to the contact is

$$
F_S = \int \left(-\lambda_1(\mathbf{n}) \left[\Psi_{s-}\Psi_{1+}^* + \Psi_{s-}^*\Psi_{1+} \right] - \lambda_2(\mathbf{n}) \left[\Psi_{s-}\Psi_{2+}^* + \Psi_{s-}^*\Psi_{2+} \right] \right) dS, \tag{5}
$$

where $\mathbf{n}$ is a unit vector orthogonal to the surface of the contact and directed from conventional superconductor to cuprate, $\lambda_1(\mathbf{n})$ and $\lambda_2(\mathbf{n})$ are the tunneling matrix elements from the first and the second pockets correspondingly. ξ is an axis parallel to $\mathbf{n}$, and Ψ_{1+}, Ψ_{2+}, and Ψ_{s-}

are values of the condensates at the contact ($\xi = \pm 0$). It is convenient to use following simple parameterization for the tunneling matrix elements

$$
\begin{aligned}
\lambda_1(\mathbf{n}) &= C(n_x^2 - n_y^2)e^{-\alpha(n_x - n_y)^2}, \\
\lambda_2(\mathbf{n}) &= C(n_x^2 - n_y^2)e^{-\alpha(n_x + n_y)^2},
\end{aligned}
\tag{6}
$$

where C and α are some parameters depending on the tunneling probability, the shape of the Fermi surface, etc. The axes x and y are directed along crystal axes a and b of the cuprate. We stress that eq.(6) is just a parameterization having correct symmetry properties. Let us introduce also the d- and g-wave tunneling matrix elements: $\lambda_d(\mathbf{n}) = \lambda_1(\mathbf{n}) + \lambda_2(\mathbf{n})$ and $\lambda_g(\mathbf{n}) = \lambda_1(\mathbf{n}) - \lambda_2(\mathbf{n})$. The symmetries of these matrix elements shown at Fig.2 are the same as the symmetries of d- and g-wave gaps in the momentum space, Fig. 1b,c. After variation of the total free energy $F = F_V^{dg} + F_V^s + F_S$ with respect to $\Psi_1^*, \Psi_2^*, \Psi_s^*$ and vector potential $\mathbf{A}$ we find: the GL equations for conventional and high-T_c superconductors

$$
\begin{aligned}
-\frac{\hbar^2}{2m_s^\star}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)^2 \Psi_s - a_s\Psi_s + 2b_s |\Psi_s|^2 \Psi_s &= 0, \\
-\frac{\hbar^2}{2m^\star}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)^2 \Psi_1 - a\Psi_1 + 2b |\Psi_1|^2 \Psi_1 + \gamma\Psi_2 &= 0, \\
-\frac{\hbar^2}{2m^\star}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)^2 \Psi_2 - a\Psi_2 + 2b |\Psi_2|^2 \Psi_2 + \gamma\Psi_1 &= 0,
\end{aligned}
\tag{7}
$$

the boundary conditions for these equations

$$
\begin{aligned}
-\frac{\hbar^2}{2m_s^\star}\mathbf{n}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\Psi_s\bigg|_{\xi=-0} &= \lambda_1(\mathbf{n})\Psi_{1+} + \lambda_2(\mathbf{n})\Psi_{2+}, \\
\frac{\hbar^2}{2m^\star}\mathbf{n}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\Psi_1\bigg|_{\xi=+0} &= \lambda_1(\mathbf{n})\Psi_{s-}, \\
\frac{\hbar^2}{2m^\star}\mathbf{n}\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\Psi_2\bigg|_{\xi=+0} &= \lambda_2(\mathbf{n})\Psi_{s-},
\end{aligned}
\tag{8}
$$

and the currents in the cuprate and conventional superconductors

$$
\begin{aligned}
\mathbf{j} &= -\frac{\hbar i e}{m^\star}\left(\Psi_1^*\nabla\Psi_1 - \Psi_1\nabla\Psi_1^* + \Psi_2^*\nabla\Psi_2 - \Psi_2\nabla\Psi_2^*\right) - \frac{4e^2}{cm^\star}\mathbf{A}(|\Psi_1|^2 + |\Psi_2|^2), \\
\mathbf{j} &= -\frac{\hbar i e}{m_s^\star}\left(\Psi_s^*\nabla\Psi_s - \Psi_s\nabla\Psi_s^*\right) - \frac{4e^2}{cm_s^\star}\mathbf{A}|\Psi_s|^2 .
\end{aligned}
\tag{9}
$$

Substituting the boundary conditions (8) into eqs. (9) we find the expression for supercurrent through the contact

$$
j = \frac{2ie}{\hbar}\lambda_1(\mathbf{n})[\Psi_{s-}\Psi_{1+}^* - \Psi_{s-}^*\Psi_{1+}] + \frac{2ie}{\hbar}\lambda_2(\mathbf{n})[\Psi_{s-}\Psi_{2+}^* - \Psi_{s-}^*\Psi_{2+}].
\tag{10}
$$

We will consider the distances larger than the cuprate superconducting correlation length. At these distances the magnitudes $|\Psi_1|$ and $|\Psi_2|$ are equal to the constant given by eq.(3), and only the phases of condensates are ξ dependent. The Josephson current through the contact can be written as

$$
j = \frac{4e}{\hbar}|\Psi||\Psi_s|\left[\lambda_1(\mathbf{n})\sin(\phi_{1+} - \phi_{s-}) + \lambda_2(\mathbf{n})\sin(\phi_{2+} - \phi_{s-})\right],
\tag{11}
$$

where ϕ_{s-}, ϕ_{1+} and ϕ_{2+} are values of the phases at the contact ($\xi = \pm 0$). We consider the situation without an external magnetic field in the contact. The magnetic field caused by the

supercurrent is very small, therefore we set $\mathbf{A} = 0$ and reduce eqs.(7) for Ψ_1 and Ψ_2 to ones for the phases

$$\frac{\hbar^2}{2m^\star} \frac{d^2 \Delta\phi}{d\xi^2} + i \frac{\hbar^2}{m^\star} \frac{d\phi}{d\xi} \frac{d\Delta\phi}{d\xi} + 2\gamma \sin \Delta\phi = 0, \tag{12}$$

$$\frac{\hbar^2}{m^\star} \left[\left(\frac{d\phi}{d\xi}\right)^2 + \frac{1}{4} \left(\frac{d\Delta\phi}{d\xi}\right)^2 \right] - i \frac{\hbar^2}{m^\star} \frac{d^2\phi}{d\xi^2} + 2 |\gamma| \left(1 + \frac{\gamma}{|\gamma|} \cos \Delta\phi\right) = 0,$$

with $\phi = (\phi_1 + \phi_2)/2$ and $\Delta\phi = \phi_2 - \phi_1$. The equilibrium value of $\Delta\phi$ is $\Delta\phi_0$ with $\Delta\phi_0 = 0$ at $\gamma < 0$ (d-wave), and $\Delta\phi_0 = \pi$ at $\gamma > 0$ (g-wave). If the current is small we can linearize eqs.(12) with respect to $\Delta\phi - \Delta\phi_0$. This gives that $\phi = (\phi_1 + \phi_2)/2 = $ const, and

$$\Delta\phi = \phi_2 - \phi_1 = \Delta\phi_0 + A \exp\left(-\xi/l_\gamma\right), \tag{13}$$

where $l_\gamma = \hbar/\sqrt{4 |\gamma| m^\star}$. It is convenient to rewrite the supercurrent (11) in the form

$$j = \frac{4e}{\hbar} |\Psi| |\Psi_s| \left[\lambda_d \sin\theta \cos \frac{\Delta\phi_c}{2} - \lambda_g \cos\theta \sin \frac{\Delta\phi_c}{2}\right], \tag{14}$$

where $\theta = \phi - \phi_{s-}$ and $\Delta\phi_c$ is the value of $\Delta\phi$ at the contact: $\Delta\phi_c = A$ for $\gamma < 0$ (d-wave) and $\Delta\phi_c = \pi + A$ for $\gamma > 0$ (g-wave). The solution we found depends on the two arbitrary constants θ and A. If the supercurrent through the contact is fixed then the eq. (14) gives one relation between these constants. Second relation follows from the minimum of total free energy. After substitution of the solution (13) into expressions (1),(2), (5) we find the part of the total free energy depending on θ and A

$$F_\Omega = |\Psi|^2 K_\gamma A^2 - 2 |\Psi| |\Psi_s| \left[\lambda_d \cos\theta \cos \frac{\Delta\phi_c}{2} + \lambda_g \sin\theta \sin \frac{\Delta\phi_c}{2}\right]. \tag{15}$$

Here $K_\gamma = l_\gamma |\gamma| = \hbar\sqrt{|\gamma|/m^*}/2$.

Let us consider at first the contact plane to be parallel to the axis a or axis b of the cuprate ($\mathbf{n} = (\pm 1, 0)$ or $\mathbf{n} = (0, \pm 1)$). In this case $\lambda_g = 0$ and $\lambda_d \neq 0$. If the ground state pairing has d-wave symmetry ($\gamma < 0$) we find from eqs. (14) and (15) the following condition of minimum

$$\left.\frac{dF_\Omega}{dA}\right|_{j=const} = \left(2K_\gamma |\Psi|^2 + \frac{\lambda_d |\Psi_s \Psi|}{2\cos\theta}\right) A = 0. \tag{16}$$

Therefore $A = 0$, and θ is determined by the usual relation

$$j = j_{cd} \sin\theta, \quad j_{cd} = \left|\frac{4e}{\hbar} \Psi \Psi_s \lambda_d\right|. \tag{17}$$

If the ground state pairing has g-wave symmetry ($\gamma > 0$) the condition of minimum is

$$\left.\frac{dF_\Omega}{dA}\right|_{j=const} = \frac{\lambda_d |\Psi_s|}{\cos\theta} + 2AK_\gamma |\Psi| = 0. \tag{18}$$

This gives

$$A = -\frac{\lambda_d |\Psi_s|}{2\cos\theta K_\gamma |\Psi|}, \tag{19}$$

and the supercurrent is

$$j = \frac{e\lambda_d^2 |\Psi_s|^2}{\hbar K_\gamma} \tan\theta. \tag{20}$$

4

One has to remember that the above equations are derived assuming that $A \ll 1$ and therefore they are not valid when θ is approaching $\pi/2$. So the picture is that deep inside the cuprate we have g-wave pairing, but in the layer of width l_γ near the contact there is admixture of the d-wave component. If current is increasing the admixture is increasing too. So the current pumps the g-wave into the d-wave in the layer of width l_γ. At critical current there is only the d-wave at the contact. Therefore the critical current in this case is exactly the same as one for the d-wave ground state: $j_c = j_{cd}$.

At an arbitrary orientation of contact plane with respect to the crystal axes both λ_d and λ_g do not vanish. Nevertheless the d-wave tunneling probability is always larger than the g-wave one: $|\lambda_d(\mathbf{n})| > |\lambda_g(\mathbf{n})|$. It is obvious from eqs.(6). Let us rewrite the current (14) as

$$j = \frac{2\sqrt{2}e}{\hbar} |\Psi| |\Psi_s| \sqrt{\lambda_d^2 + \lambda_g^2 + (\lambda_d^2 - \lambda_g^2) \cos \Delta\phi_c} \cdot \sin(\theta - \alpha), \tag{21}$$

where $\sin \alpha = \sqrt{2}\lambda_g \sin \frac{\Delta\phi_c}{2} \Big/ \sqrt{\lambda_d^2 + \lambda_g^2 + (\lambda_d^2 - \lambda_g^2) \cos \Delta\phi_c}$, and let us introduce the notations $j_{cd} = \left| \frac{4e}{\hbar} \Psi\Psi_s\lambda_d \right|$, $j_{cg} = \left| \frac{4e}{\hbar} \Psi\Psi_s\lambda_g \right|$, keeping in mind that $j_{cd} > j_{cg}$. Let us assume first that the ground state of the cuprate has the d-wave gap ($\gamma < 0$). Then at any supercurrent the system is in the d-wave everywhere including the contact ($\Delta\phi_c = 0$), and from eq.(21) we conclude that the critical current is $j_c = j_{cd}$. Consider now the situation when the ground state of the cuprate has the g-wave symmetry ($\gamma > 0$). Then at $j < j_{cg}$ the system remains in g-wave state everywhere including the contact, $\Delta\phi_c = \pi$. Increasing the current above j_{cg} we start to pump the g-wave into the d-wave in the surface layer of the width l_γ. When the current reaches j_{cd}, according to eq.(21) the phase takes the value $\Delta\phi_c = 0$, and this is the critical point. So at the critical point in the contact the g-wave is completely pumped into the d-wave, and the critical current is exactly the same as in the case of the d-wave pairing: $j_c = j_{cd}$.

4 SQUID

The g-wave pairing can be revealed in the SQUID. Consider at first the geometry of the SQUID experiment [14], results of which are interpreted as a very strong evidence in favor of the d-wave pairing. The setup is shown schematically at Fig.3. Faces of the superconducting corner are parallel to crystal axes $\mathbf{a}$ and $\mathbf{b}$ of the cuprate superconductor so that $\lambda_g(\mathbf{n}) = 0$ and $\lambda_d(\mathbf{n})$ have opposite signs at the tunnel contacts: $\lambda_{d1} = -\lambda_{d2} = \lambda_d$. The indexes 1,2 numerate the contacts. Using eqs.(14),(15) the total current and the free energy can be written as

$$j = j_1 + j_2 = \frac{4e}{\hbar} |\Psi| |\Psi_s| \lambda_d \left[\sin \theta_1 \cos \frac{\Delta\phi_{c1}}{2} - \sin \theta_2 \cos \frac{\Delta\phi_{c2}}{2} \right], \tag{22}$$

$$F_\Omega = |\Psi|^2 K_\gamma (A_1^2 + A_2^2) - 2 |\Psi| |\Psi_s| \lambda_d \left[\cos \theta_1 \cos \frac{\Delta\phi_{c1}}{2} - \cos \theta_2 \cos \frac{\Delta\phi_{c2}}{2} \right]. \tag{23}$$

Here θ_i, A_i, and $\Delta\phi_{ci} = \Delta\phi_0 + A_i$ (i=1,2) are the quantities introduced in previous section. Quantization condition is standard

$$\theta_1 = \theta + \frac{\pi}{2} - \pi\frac{\Phi}{\Phi_0}, \quad \theta_2 = \theta + \frac{\pi}{2} + \pi\frac{\Phi}{\Phi_0} - 2\pi m, \tag{24}$$

where $\Phi_0 = \pi c\hbar/e$ is the quantum of magnetic flux, and m is an integer number.

Let us consider at first the case $\gamma < 0$ which corresponds to the d-wave pairing in the cuprate ground state. Minimum of the free energy (23) at fixed supercurrent (22) is defined by the equations

$$\left.\frac{\partial F_\Omega}{\partial A_i}\right|_{j=const} = \left(2K_\gamma |\Psi|^2 + \frac{\lambda_d |\Psi\Psi_s| \sin(\pi\Phi/\Phi_0)}{2\cos\theta} \right) A_i = 0. \tag{25}$$

The solution of these equations is $A_1 = A_2 = 0$, so the system remains in the d-wave state everywhere including the contacts, and the current equals

$$j = j_{cd} \sin \theta, \quad j_{cd} = \frac{8e}{\hbar} |\Psi\Psi_s\lambda_d| \left|\sin\left(\pi\frac{\Phi}{\Phi_0}\right)\right|. \tag{26}$$

The plot of j_{cd} is represented at Fig.4a and is in agreement with experimental data [13].
Now consider the case $\gamma > 0$ which corresponds to the g-wave pairing in the cuprate ground state. Minimum of the free energy (23) at fixed supercurrent (22) is defined by the equations

$$\left.\frac{\partial F_\Omega}{\partial A_1}\right|_{j=const} = \left[2A_1 K_\gamma |\Psi|^2 + \lambda_d \cdot \frac{A_1 - A_2 \cos(\theta_1 - \theta_2)}{A_1 \cos\theta_1 - A_2 \cos\theta_2} |\Psi_s\Psi|\right] = 0, \tag{27}$$

$$\left.\frac{\partial F_\Omega}{\partial A_2}\right|_{j=const} = \left[2A_2 K_\gamma |\Psi|^2 + \lambda_d \cdot \frac{A_2 - A_1 \cos(\theta_1 - \theta_2)}{A_1 \cos\theta_1 - A_2 \cos\theta_2} |\Psi_s\Psi|\right] = 0.$$

There are two solutions of these equations:

$$symmetric: \quad A_1 = A_2 = -\frac{\lambda_d}{2K_\gamma}\frac{|\Psi_s|}{|\Psi|}\frac{\sin(\pi\Phi/\Phi_0)}{\cos\theta}, \tag{28}$$

$$antisymmetric: \quad A_1 = -A_2 = \frac{\lambda_d}{2K_\gamma}\frac{|\Psi_s|}{|\Psi|}\frac{\cos(\pi\Phi/\Phi_0)}{\sin\theta}.$$

The supercurrents and free energies corresponding to symmetric and antisymmetric solutions are

$$j^{(+)} = \frac{2e\Lambda_d}{\hbar}\sin^2\left(\pi\frac{\Phi}{\Phi_0}\right)\tan\theta, \qquad F_\Omega^{(+)} = \frac{\Lambda_d}{2}\left[-\sin^2\left(\pi\frac{\Phi}{\Phi_0}\right) + \frac{2j^2}{(2e\Lambda_d/\hbar)^2\sin^2\left(\pi\frac{\Phi}{\Phi_0}\right)}\right] \tag{29}$$

$$j^{(-)} = -\frac{2e\Lambda_d}{\hbar}\cos^2\left(\pi\frac{\Phi}{\Phi_0}\right)\text{ctan}\theta, \qquad F_\Omega^{(-)} = \frac{\Lambda_d}{2}\left[-\cos^2\left(\pi\frac{\Phi}{\Phi_0}\right) + \frac{2j^2}{(2e\Lambda_d/\hbar)^2\cos^2\left(\pi\frac{\Phi}{\Phi_0}\right)}\right],$$

where $\Lambda_d = \lambda_d^2|\Psi_s|^2/K_\gamma$. The indexes (+), (-) denote symmetric and antisymmetric solutions correspondingly. The physical state corresponds to minimum of the free energy and therefore at $-1/4 + m < \Phi/\Phi_0 < 1/4 + m$ the SQUID is in the antisymmetric state ($A_1 = -A_2$), and at $1/4 + m < \Phi/\Phi_0 < 3/4 + m$ it is in the symmetric state ($A_1 = A_2$), m is an integer. One has to remember that eqs. (28) and (29) are derived at $A_i \ll 1$, and they are not valid when θ is approaching $\pi/2$ for the symmetric solution and when θ is approaching 0 for the antisymmetric solution. So at $\gamma > 0$ inside the superconductor we have g-wave pairing, but in layers of width l_γ near the contacts there is admixture of the d-wave component. If the current is increasing the admixture is increasing too. So the current pumps the g-wave into the d-wave state in the layer of width l_γ. At the critical current there is only the d-wave at the contact. The critical currents corresponding to symmetric and antisymmetric solutions can be easily found from eq.(22) if we substitute $A_1 = A_2 = \pi$ or $A_1 = -A_2 = \pi$.

$$j_c^{(+)} = \frac{8e}{\hbar}\left|\Psi\Psi_s\lambda_d \sin\left(\pi\frac{\Phi}{\Phi_0}\right)\right|, \tag{30}$$

$$j_c^{(-)} = \frac{8e}{\hbar}\left|\Psi\Psi_s\lambda_d \cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|.$$

The real critical current is $j_{cg} = \max\left\{j_c^{(+)}, j_c^{(-)}\right\}$ The plot of j_{cg} as a function of magnetic flux is given at Fig.4b. It is interesting that it has a period $\Phi_0/2$ which could be naively interpreted as flux quantization with an effective charge $4e$.

So we see that the dependence of the SQUID critical current on the magnetic flux is different for different ground states of the cuprate superconductor. We considered above the geometry with contact planes parallel to crystal axes **a** and **b** of the cuprate. If the bulk ground state of the cuprate has d-wave pairing ($\gamma < 0$) the system remains in the d-wave everywhere including the contacts and the critical current is proportional to $|\sin{(\pi\Phi/\Phi_0)}|$. If the bulk ground state of the cuprate has g-wave pairing ($\gamma > 0$) then the current pumps the g-wave into the d-wave in the layers of the width l_γ near the contacts. The critical current in this case has a very unusual dependence on the magnetic flux presented at Fig.4b.

Let us consider now a more general geometry of SQUID contacts when $\lambda_g \neq 0$. We assume that it is a straight angle rotated with respect to crystal axes of cuprate, therefore $\lambda_{d1} = -\lambda_{d2} = \lambda_d$, $\lambda_{g1} = \lambda_{g2} = \lambda_g$, see Fig.5. We assume also that the angle of rotation is not very small, so that $\lambda_d \sim \lambda_g$. Consider at first the case of the d-wave ground state of the cuprate superconductor ($\gamma < 0$, $\Delta\phi_0 = 0$). The free energy is given by the sum of eqs.(15) corresponding to the two contacts. The same is valid for supercurrent (14). Minimization of the free energy at fixed supercurrent with respect to variation of phases A_i gives in this case a unique solution. At $A_i \ll 1$ the solution can be found analytically

$$A_1 = A_2 = \frac{\lambda_g}{2K_\gamma} \frac{|\Psi_s|}{|\Psi|} \frac{\cos{(\pi\Phi/\Phi_0)}}{\cos\theta}. \tag{31}$$

So at $\lambda_g \neq 0$ external current pumps the d-wave into the g-wave in the layer of width l_γ near the contacts. Keeping in mind that $A_1 = A_2 = A$ we can write the supercurrent at arbitrary A in the form

$$j = j_c \cdot \cos(\alpha + A/2) \cdot \sin\theta, \tag{32}$$

with $\sin\alpha = \lambda_g \cos(\pi\frac{\Phi}{\Phi_0}) \big/ \sqrt{\lambda_d^2 \sin^2(\pi\frac{\Phi}{\Phi_0}) + \lambda_g^2 \cos^2(\pi\frac{\Phi}{\Phi_0})}$, and the critical current given by

$$j_c = \frac{8e}{\hbar}|\Psi\Psi_s|\sqrt{\lambda_d^2 \sin^2(\pi\frac{\Phi}{\Phi_0}) + \lambda_g^2 \cos^2(\pi\frac{\Phi}{\Phi_0})}. \tag{33}$$

The plot of the critical current is given at Fig.4c. The admixture of the g-wave at the contact at critical current is defined by α. It is equal to 100% at $\Phi = 0$, and it is equal to 0 at $\Phi = \Phi_0/2$.

Now we consider the last situation: geometry with $\lambda_{d1} = -\lambda_{d2} = \lambda_d$, $\lambda_{g1} = \lambda_{g2} = \lambda_g \neq 0$ (see Fig.5) and the g-wave ground state of the cuprate superconductor ($\gamma > 0$, $\Delta\phi_0 = \pi$). Minimization of the free energy shows that only one solution exists, which at small A_i has the form

$$A_1 = A_2 = -\frac{\lambda_d}{2K_\gamma} \frac{|\Psi_s|}{|\Psi|} \frac{\sin{(\pi\Phi/\Phi_0)}}{\cos\theta}. \tag{34}$$

It is similar to the symmetric solution (28) for the geometry with $\lambda_g = 0$. It is interesting that in this case there is no an analog of the antisymmetric solution (28). So, as usual the external current pumps the g-wave into the d-wave at the contact surfaces. The supercurrent at an arbitrary A_i, but with $A_1 = A_2 = A$ can be written as

$$j = j_c \cdot \sin(\alpha + A/2) \cdot \sin\theta, \tag{35}$$

with exactly the same j_c and α as those for the d-wave ground state, given by eqs. (32), (33). The admixture of the d-wave at the contact at critical current is defined by α. It is equal to 100% at $\Phi = \Phi_0/2$ and it is equal 0 at $\Phi = 0$. So the critical current through the SQUID is given by eq.(33) and it is independent of whether we have the d-wave or the g-wave pairing in ground state. The plot of the critical current is given at Fig.4c.

The following question arises. In the case of contact planes parallel to the crystal axes **a** and **b** the dependences of critical current on flux are different for different ground states: the

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dependence is given by Fig. 4a for the d-wave ground state, and it is given by Fig. 4b for the g-wave ground state. On the other hand as soon as the contact is rotated (Fig. 5) the dependence is the same for the d- and g-wave ground states, and it is given at Fig. 4c. What is the angle of rotation necessary to change the regime? The origin of the difference in regimes is in the fact that at $\lambda_g = 0$ there are both symmetric and antisymmetric solutions, see eq. (28), but at finite λ_g the anisymmetric solution disappears, see eq. (34). One can prove that with increasing λ_g from zero the free energy corresponding to the antisymmetric solution is increasing very fast, and at $\lambda_g / \lambda_d > \beta_c$,

$$\beta_c \sim \frac{\lambda_d}{2K_\gamma} \frac{|\Psi_s|}{|\Psi|} \ll 1, \tag{36}$$

it becomes very high. So the regime is changed when the rotation angle β is larger than β_c. The value of β_c is very small because it is proportional to the tunneling amplitude.

5 Conclusions

We have considered the scenario with the small Fermi surface consisting of the hole pockets. The picture can be relevant to underdoped cuprate superconductors. The small Fermi surface together with mechanism of the magnetic pairing results in the possibility of having both the d- and the g-wave pairing. Energy splitting between these states is small. The ground state symmetry is defined by the interplay between the magnetic pairing and Coulomb repulsion. We have formulated an effective Ginsburg-Landau theory describing this situation.

We demonstarate that in the Josephson junction or the SQUID consisting of cuprate super-conductor and conventional superconductor the supercurrent pumps the d-wave into g-wave (or the g-wave into d-wave) in the thin layer near the contact. This pumping influences the SQUID interference picture. If the bulk ground state has d-wave symmetry and contact planes are parallel to crystal axes **a** and **b** of the cuprate the dependence of the SQUID critical current on the magnetic flux is shown at Fig.4a. This is a well known picture for the d-wave superconductor. If the bulk ground state has g-wave symmetry and contact planes are parallel to crystal axes **a** and **b** of the cuprate the dependence of the SQUID critical current on the magnetic flux is shown at Fig.4b. It is very unusual and could be naively interpreted as flux quantization with an effective charge $4e$. If the contact planes are rotated with respect to crystal axes **a** and **b** at the angle $\beta > \beta_c$, then the SQUID interference picture is the same for the d- and g-wave pairing in the bulk ground state, and this picture is presented at Fig. 4c. The angle β_c given by eq.(36) is very small, since it is proportional to the tunneling amplitude.

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FIIGURE CAPTIONS

Fig. 1. **a.** Fermi surface in magnetic Brillouin zone which is equivalent to the two-poket Fermi surface (dashed line). **b.** Symmetry of the d-wave pairing in momentum space. **c.** Symmetry of the g-wave pairing in momentum space.

Fig. 2. **a.** The d-wave tunneling amplitude. **b.** The g-wave tunneling amplitude, n_x and n_y are crystal axes of cuprate.

Fig. 3. Geometry of the SQUID experiment with contact planes parallel to the crystal axes.

Fig. 4. Dependence of the SQUID critical current on magnetic flux. **a.** The d-wave bulk ground state and contact planes exactly parallel to the axes **a** and **b** of the cuprate. **b.** The g-wave bulk ground state and contact planes exactly parallel to the axes **a** and **b** of the cuprate. **c.** The d- or g-wave bulk ground state and superconducting corner rotated by a angle $\beta > \beta_c$ with respect to the crystal axes **a** and **b**.

Fig. 5. Geometry of the SQUID experiment with the superconducting corner rotated by an angle β with respect to crystal axes.
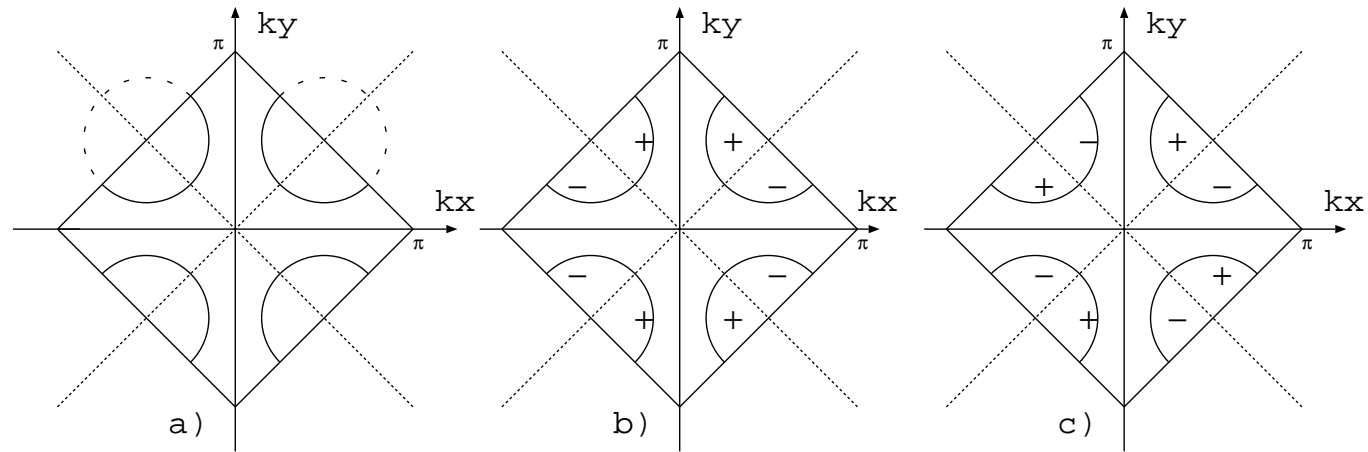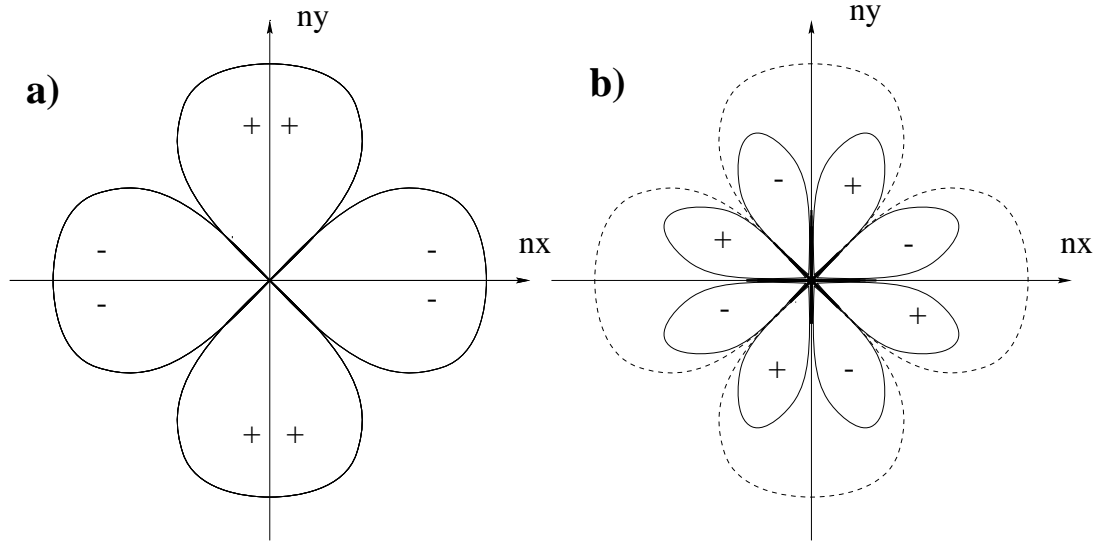
Figure 1:

Figure 2:

1-st tunnel contact

j1

j

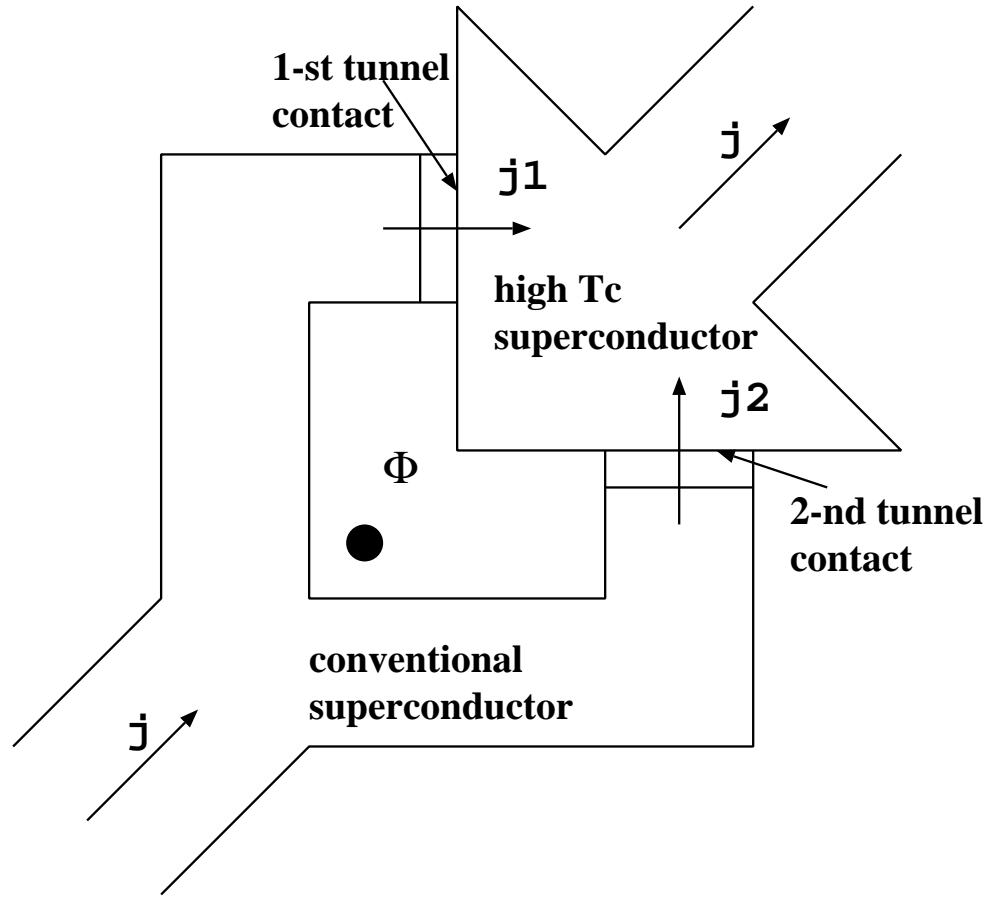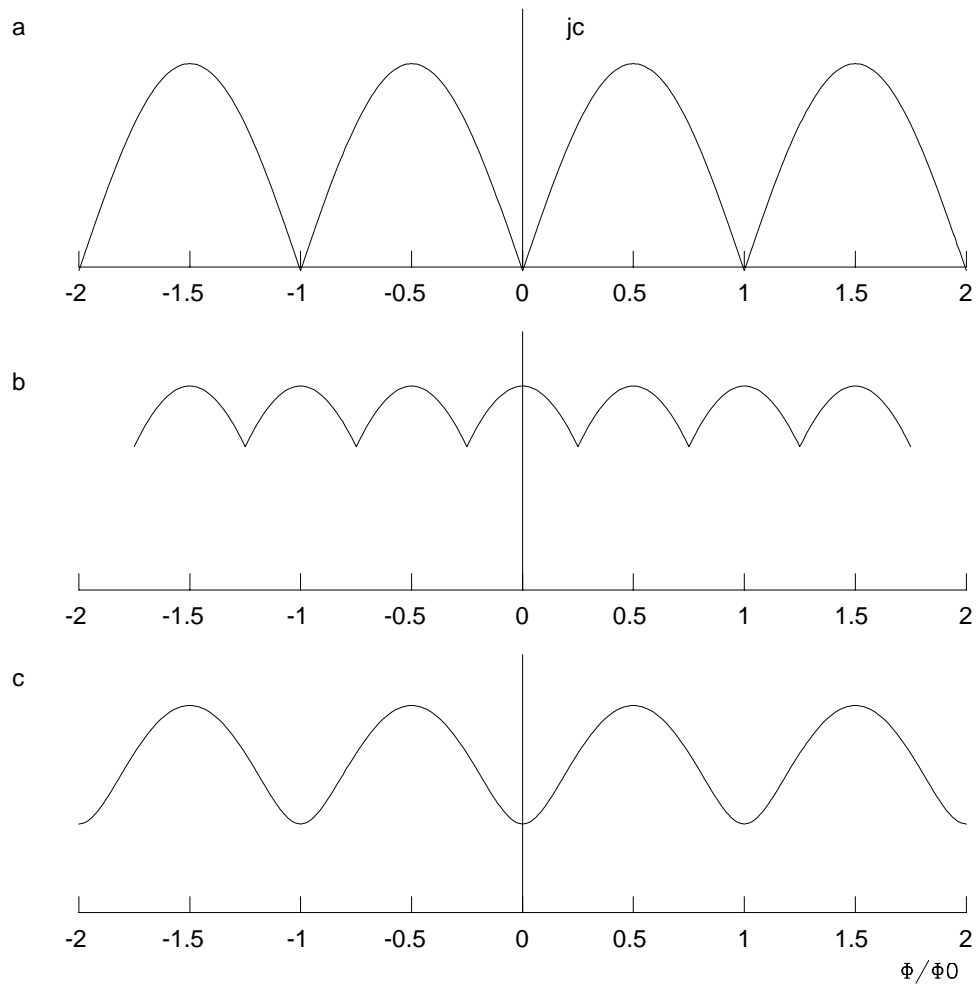**high Tc superconductor**

j2

Φ

2-nd tunnel contact

j

conventional superconductor

Figure 3:

Figure 4:

Figure 5: